

Bahasa Formal Bahasa Bebas Context



Pertemuan Ke - 10

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TIU dan TIK

1. Memahami tata bahasa bebas konteks, parsing serta penyederhanaan tata bahasa bebas konteks
2. Mampu mengerjakan soal parsing dan penyederhanaan tata bahasa bebas konteks

$\{a^n b^n : n \geq 0\}$ $\{ww^R\}$

Bahasa Regular

a^*b^* $(a+b)^*$

Bahasa Bebas Context

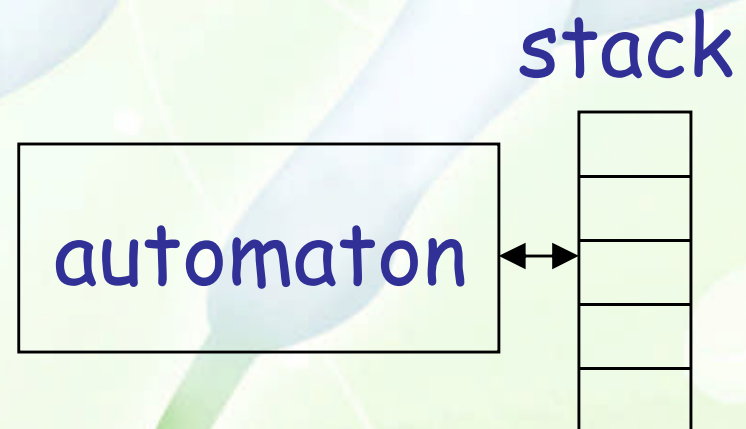
$\{a^n b^n\}$ $\{ww^R\}$

Bahasa Regular

Bahasas Bebas Konteks

Grammer bebas konteks

Pushdown Automata



Grammer Bebas Konteks



Grammer

Bahasa pengekspresian Gramer

Contoh: **bahasa Inggris**

$\langle \textit{sentence} \rangle \rightarrow \langle \textit{noun_phrase} \rangle \langle \textit{predicate} \rangle$

$\langle \textit{noun_phrase} \rangle \rightarrow \langle \textit{article} \rangle \langle \textit{noun} \rangle$

$\langle \textit{predicate} \rangle \rightarrow \langle \textit{verb} \rangle$

⟨ *article* ⟩ → *a*

⟨ *article* ⟩ → *the*

⟨ *noun* ⟩ → *cat*

⟨ *noun* ⟩ → *dog*

⟨ *verb* ⟩ → *runs*

⟨ *verb* ⟩ → *walks*

Derivasi dari “**the dog walks**”:

⟨ *sentence* ⟩ ⇒ ⟨ *noun _ phrase* ⟩ ⟨ *predicate* ⟩
⇒ ⟨ *noun _ phrase* ⟩ ⟨ *verb* ⟩
⇒ ⟨ *article* ⟩ ⟨ *noun* ⟩ ⟨ *verb* ⟩
⇒ *the* ⟨ *noun* ⟩ ⟨ *verb* ⟩
⇒ *the dog* ⟨ *verb* ⟩
⇒ *the dog walks*

Derivasi dari “a cat runs”:

$\langle sentence \rangle \Rightarrow \langle noun_phrase \rangle \langle predicate \rangle$
 $\Rightarrow \langle noun_phrase \rangle \langle verb \rangle$
 $\Rightarrow \langle article \rangle \langle noun \rangle \langle verb \rangle$
 $\Rightarrow a \langle noun \rangle \langle verb \rangle$
 $\Rightarrow a \text{ cat } \langle verb \rangle$
 $\Rightarrow a \text{ cat runs}$

Bahasa dari gramer:

$L = \{$ "a cat runs",
"a cat walks",
"the cat runs",
"the cat walks",
"a dog runs",
"a dog walks",
"the dog runs",
"the dog walks" $\}$

Notasi

Aturan Produksi

$\langle \textit{noun} \rangle \rightarrow \textit{cat}$

$\langle \textit{noun} \rangle \rightarrow \textit{dog}$

Variaber

Terminal

Contoh

Grammer:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivasi dari kalimat :

$$S \Rightarrow aSb \Rightarrow ab$$

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Apakah Gramer Berikut merupakan Bahasa

Gramer:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivasi dari kalimat : *aabb*

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivasi Lain :

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb$

$\Rightarrow aaaaSbbbb \Rightarrow aaabbbbb$

Bahasa pada gramer

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$L = \{a^n b^n : n \geq 0\}$$

Notasi Lain

$$G = (V, T, S, P)$$

Grammer

V : Himpunan variabel

T : Himpunan simbol terminal

S : Variabel awal

P : Himpunan aturan Produksi

Contoh

Grammer G :

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$G = (V, T, S, P)$$

$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow aSb, S \rightarrow \lambda\}$$

Notasi Lain

Form Sentensial:

sebuah kalimat tersiri dari variabel dan terminal

Example:

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$

Form Sentensial

Kalimat

Penulisan:

$$S \stackrel{*}{\Rightarrow} aaabbb$$

Instead of:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

Secara umum dapat ditulis : $w_1 \xRightarrow{*} w_n$

Jika : $w_1 \Rightarrow w_2 \Rightarrow w_3 \Rightarrow \dots \Rightarrow w_n$

Dengan akhir : $w \xRightarrow{*} w$



Contoh

Grammer

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivasi

*

$$S \Rightarrow \lambda$$

*

$$S \Rightarrow ab$$

*

$$S \Rightarrow aabb$$

*

$$S \Rightarrow aaabbb$$

Contoh

Grammer

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivasi

$$S \stackrel{*}{\Rightarrow} aaSbb$$

$$aaSbb \stackrel{*}{\Rightarrow} aaaaaaSbbbbbb$$

Contoh 1

Grammer : $G \quad S \rightarrow Ab$

$A \rightarrow aAb$

$A \rightarrow \lambda$



Derivasi

$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aaaAbbbb$
 $\Rightarrow aaaaAbbbbb \Rightarrow aaaabbbbb$

*

$S \Rightarrow aaaabbbbb$

*

$S \Rightarrow aaaaaabbbbbbb$

*

$S \Rightarrow a^n b^n b$

Bahasa pada Gramer

Untuk sebuah gramer G
Dimulai dengan variabel: S

$$L(G) = \{w : S \Rightarrow^* w\}$$

String pada terminal

Contoh

Untuk gramer $G : S \rightarrow Ab$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

$$L(G) = \{a^n b^n b : n \geq 0\}$$

Selama : $S \xRightarrow{*} a^n b^n b$

Notasi yang tepat

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$



$$A \rightarrow aAb \mid \lambda$$



$$\langle \textit{article} \rangle \rightarrow a$$

$$\langle \textit{article} \rangle \rightarrow \textit{the}$$



$$\langle \textit{article} \rangle \rightarrow a \mid \textit{the}$$

Contoh

Grammer Bebas kontek : G $S \rightarrow aSb$

$S \rightarrow \lambda$

derivasi:

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

Grammer Bebas Konteks: $G \quad S \rightarrow aSb$

$S \rightarrow \lambda$

Derivasi lain :

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$L(G) = \{a^n b^n : n \geq 0\}$$

Describes parentheses:

(((()))

Contoh

Grammer Bebas Konteks : G $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \lambda$

Derivasi :

$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$

Grammer Bahasa Konteks : G $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \lambda$

Derivasi Lain :

$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abaSaba \Rightarrow abaaba$

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda$$

$$L(G) = \{ww^R : w \in \{a,b\}^*\}$$

Contoh

Grammer Bahasa Konteks : G $S \rightarrow aSb$

$S \rightarrow SS$

$S \rightarrow \lambda$

Derivasi :

$S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow ab$

Grammer Bebas Konteks : $G \quad S \rightarrow aSb$

$S \rightarrow SS$

$S \rightarrow \lambda$

Derivasi :

$S \Rightarrow SS \Rightarrow aSbS \Rightarrow abS \Rightarrow abaSb \Rightarrow abab$

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \rightarrow \lambda$$

$$L(G) = \{w : n_a(w) = n_b(w), \\ \text{dan } n_a(v) \geq n_b(v) \\ \text{pada prefix lain } v\}$$

$$S \rightarrow aSb$$

$$S \rightarrow SS$$

$$S \rightarrow \lambda$$

$$L(G) = \{w : n_a(w) = n_b(w), \\ \text{dan } n_a(v) \geq n_b(v) \\ \text{pada prefix lain } v\}$$

Diskripsi sesuai

Dengan tanda

Kurung :

() ((())) (())

TEORI BAHASA OTOMATA

Definisi: **Grammer Bebas Kontek**

Grammar $G = (V, T, S, P)$

Variable

Simbol variabel
Terminal awal

Produksi dari form :

Variable $A \rightarrow x$ String dari variabel
dan terminal

$$G = (V, T, S, P)$$

$$L(G) = \{w : S \Rightarrow w, w \in T^*\}$$

Definisi

bahasa Bebas Konteks

Sebuah Bahasa L adalah bebas konteks

Jika dan hanya jika G

Grammer bebas konteks

dengan

$$L = L(G)$$

Derivasi Order

$$1. S \rightarrow AB$$

$$2. A \rightarrow aaA$$

$$4. B \rightarrow Bb$$

$$3. A \rightarrow \lambda$$

$$5. B \rightarrow \lambda$$

Derivasi dari kiri:

$$\begin{array}{ccccccccc} & 1 & & 2 & & 3 & & 4 & & 5 \\ S & \Rightarrow & AB & \Rightarrow & aaAB & \Rightarrow & aaB & \Rightarrow & aaBb & \Rightarrow & aab \end{array}$$

Derivasi dari kanan:

$$\begin{array}{ccccccccc} & 1 & & 4 & & 5 & & 2 & & 3 \\ S & \Rightarrow & AB & \Rightarrow & ABb & \Rightarrow & Ab & \Rightarrow & aaAb & \Rightarrow & aab \end{array}$$

$$S \rightarrow aAB$$

Derivasi Order $A \rightarrow bBb$

$$B \rightarrow A \mid \lambda$$

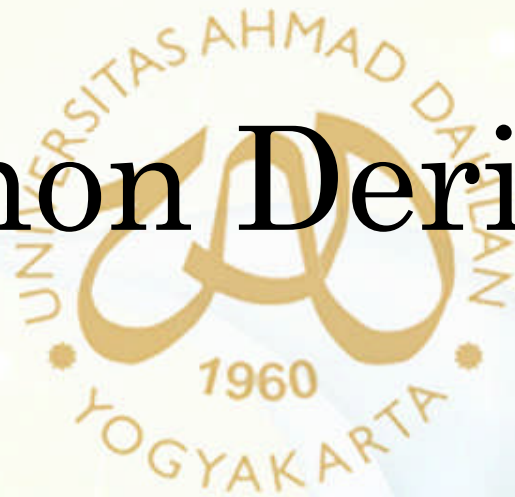
Derivasi dari kiri :

$$\begin{aligned} S &\Rightarrow aAB \Rightarrow abBbB \Rightarrow abAbB \Rightarrow abbBbbB \\ &\Rightarrow abbbbB \Rightarrow abbbb \end{aligned}$$

Derivasi dari kanan :

$$\begin{aligned} S &\Rightarrow aAB \Rightarrow aA \Rightarrow abBb \Rightarrow abAb \\ &\Rightarrow abbBbb \Rightarrow abbbb \end{aligned}$$

Pohon Derivasi

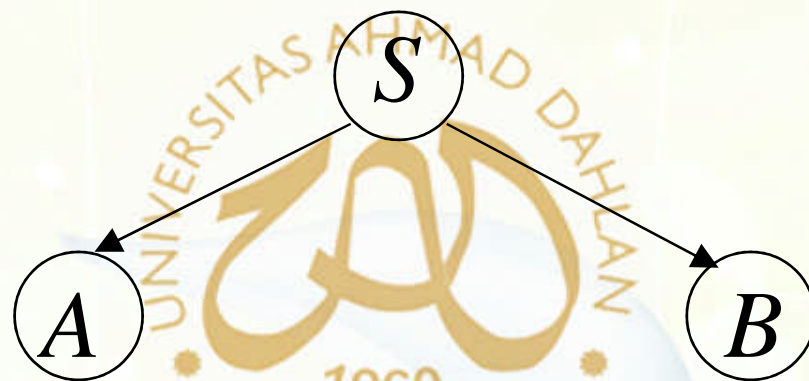


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB$$

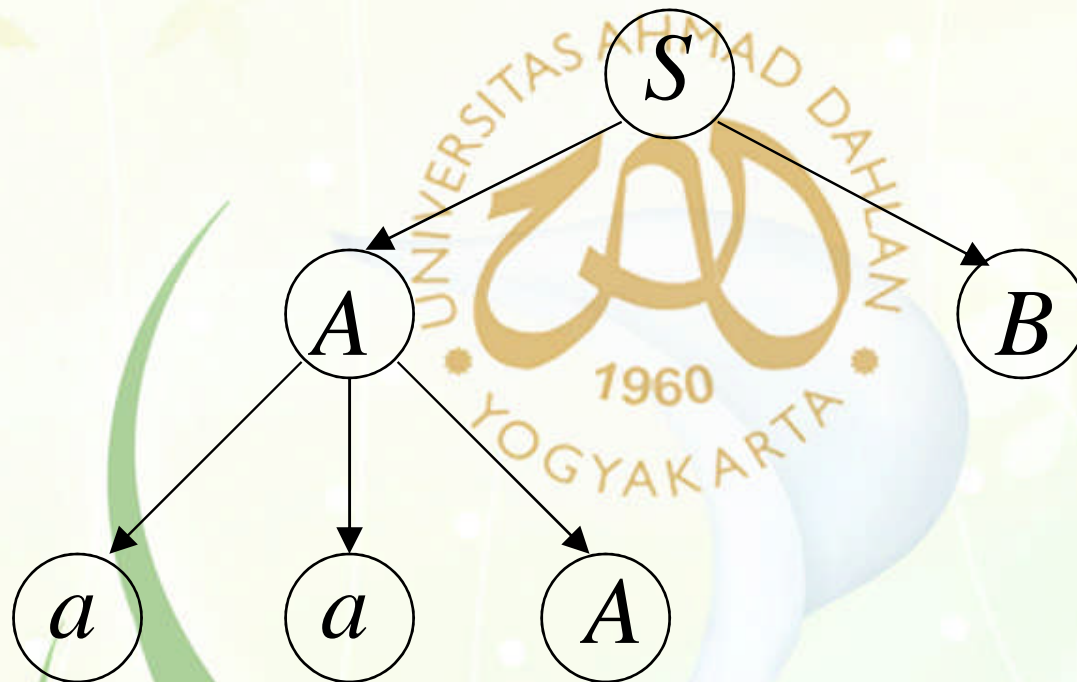


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB$$

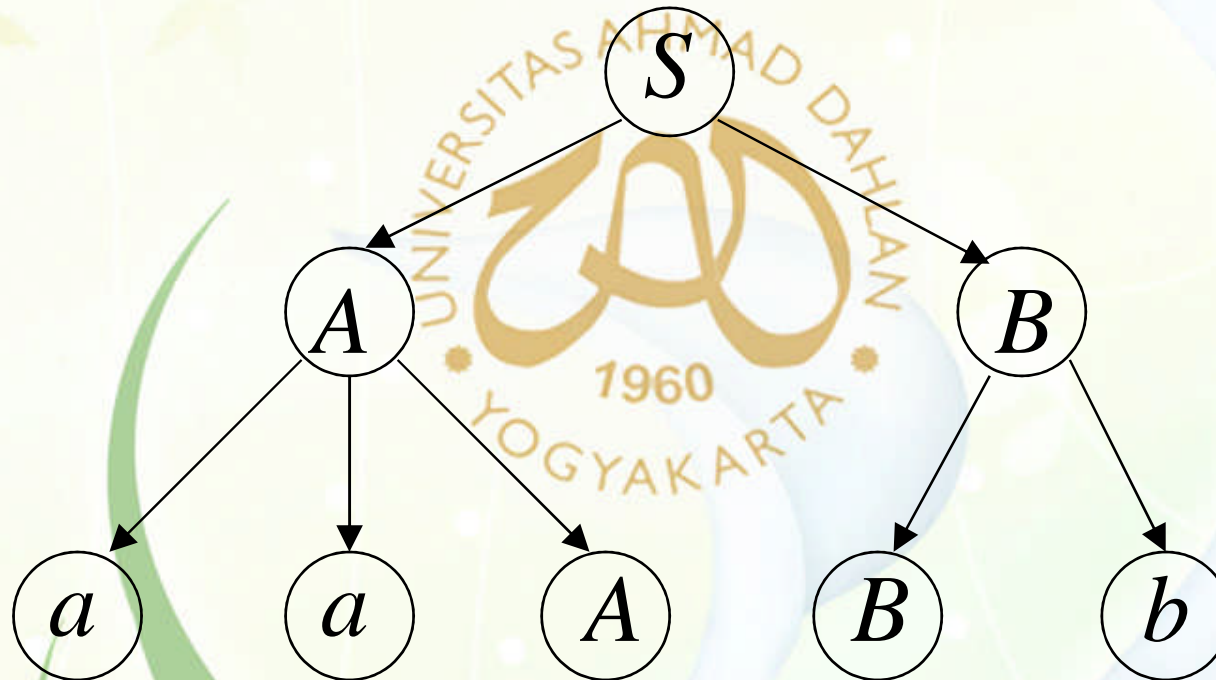


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb$$

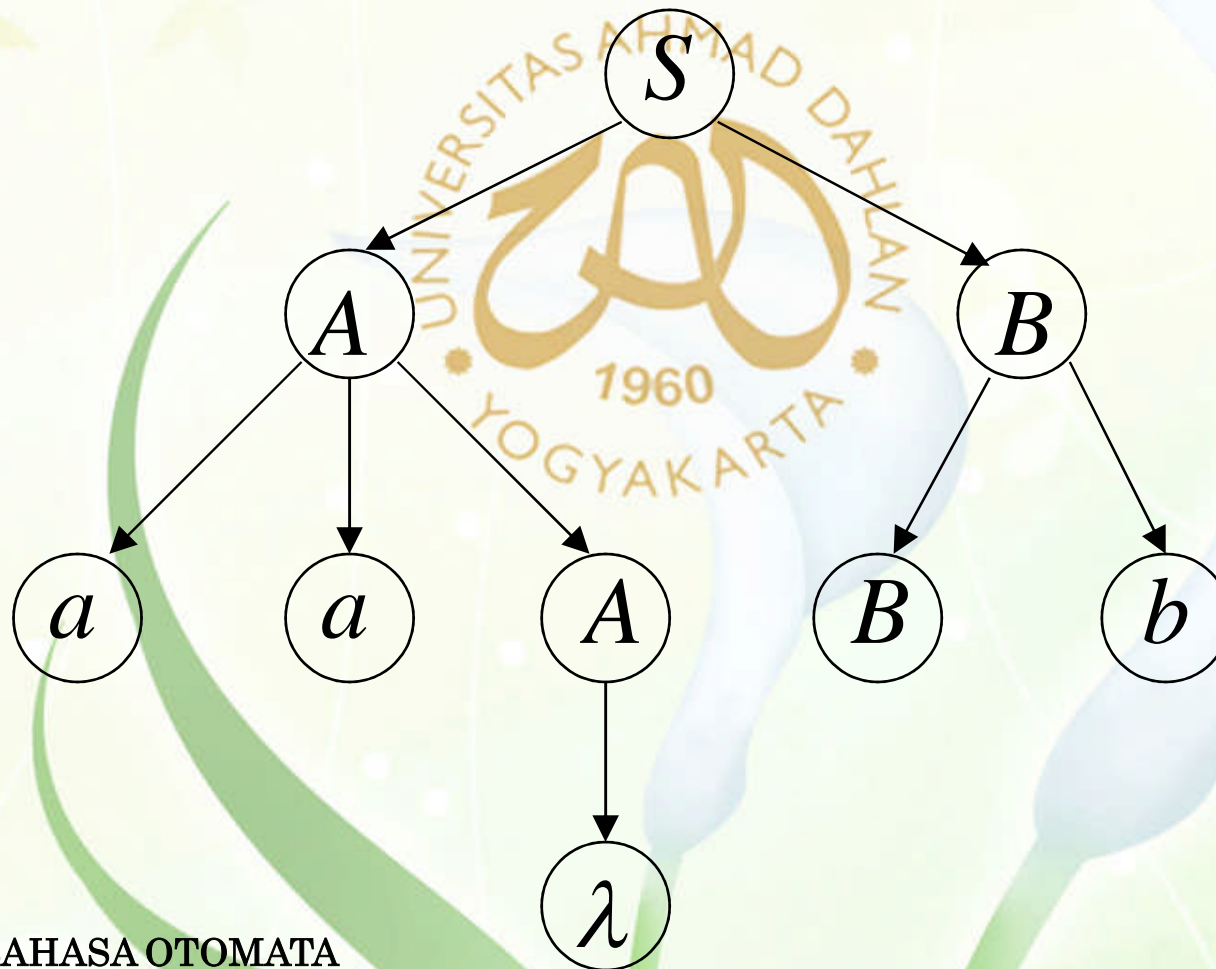


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb$$



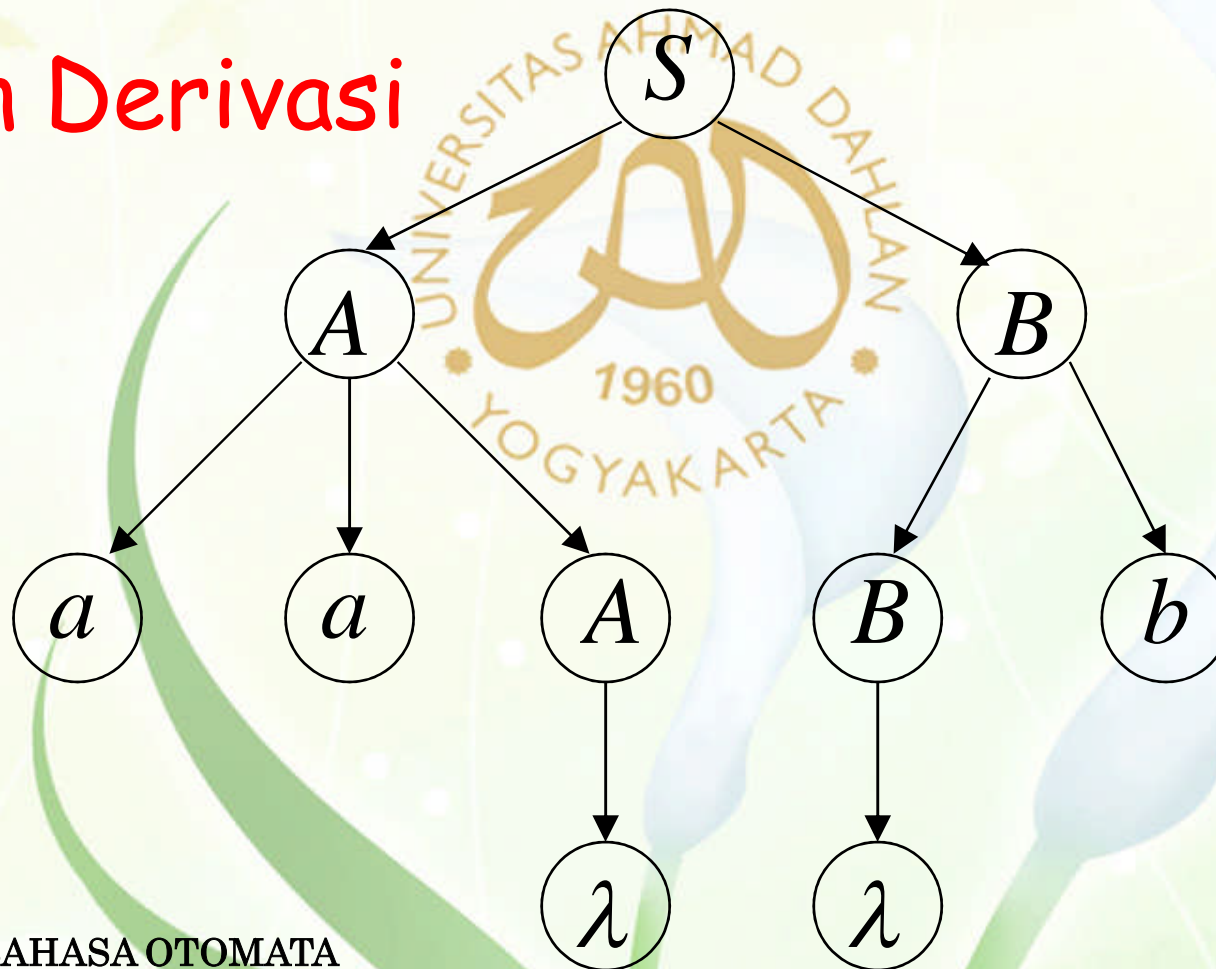
$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Pohon Derivasi



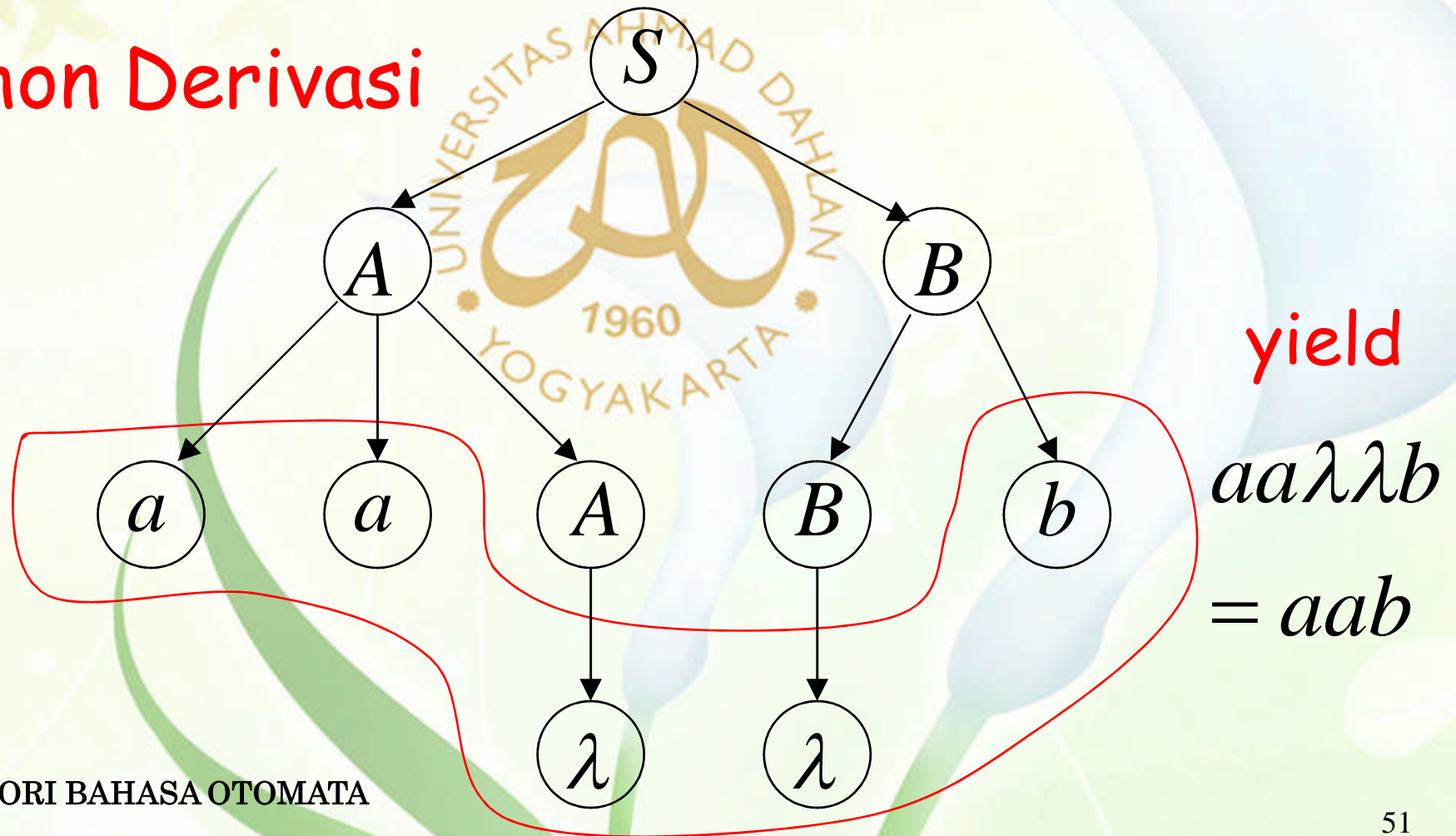
$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Pohon Derivasi



Parsial Pohon Derivasi

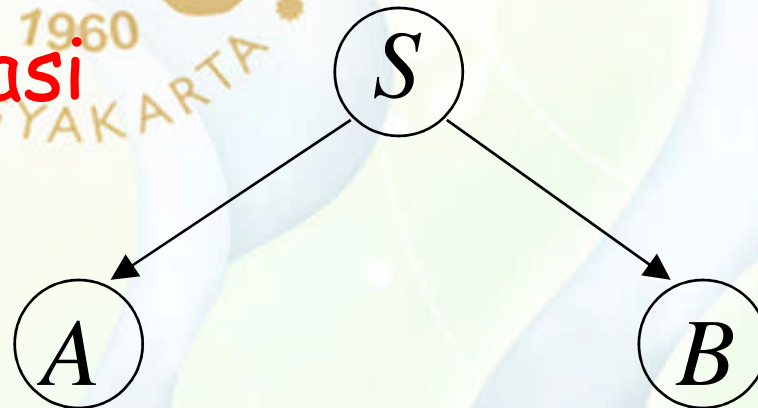
$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

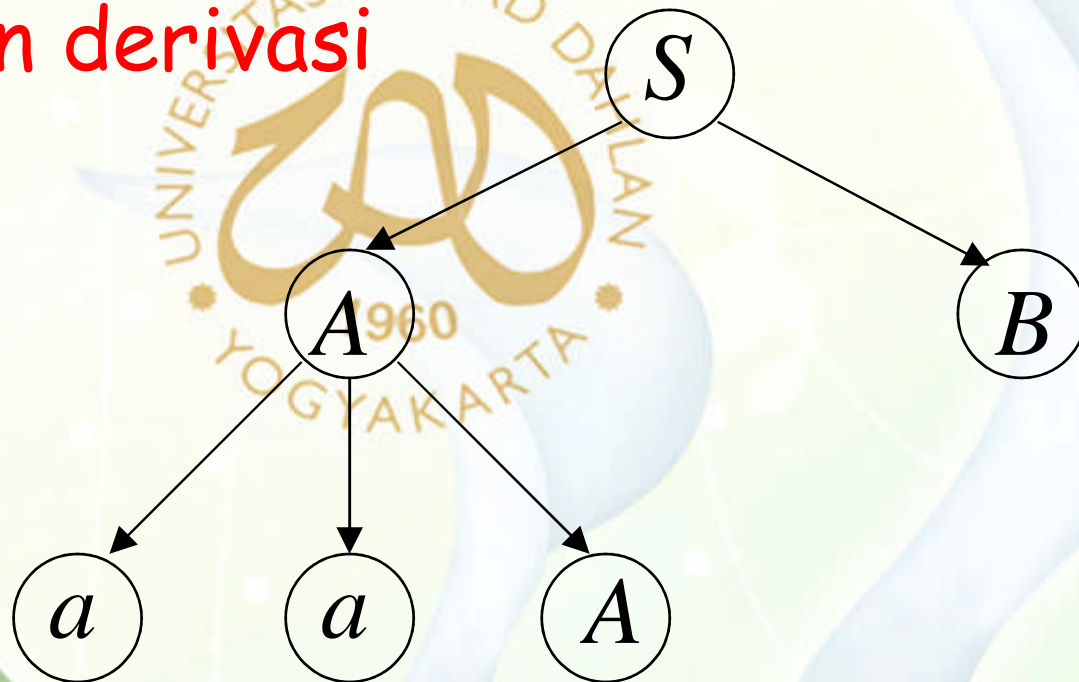
$$S \Rightarrow AB$$

Parsial pohon derivasi



$$S \Rightarrow AB \Rightarrow aaAB$$

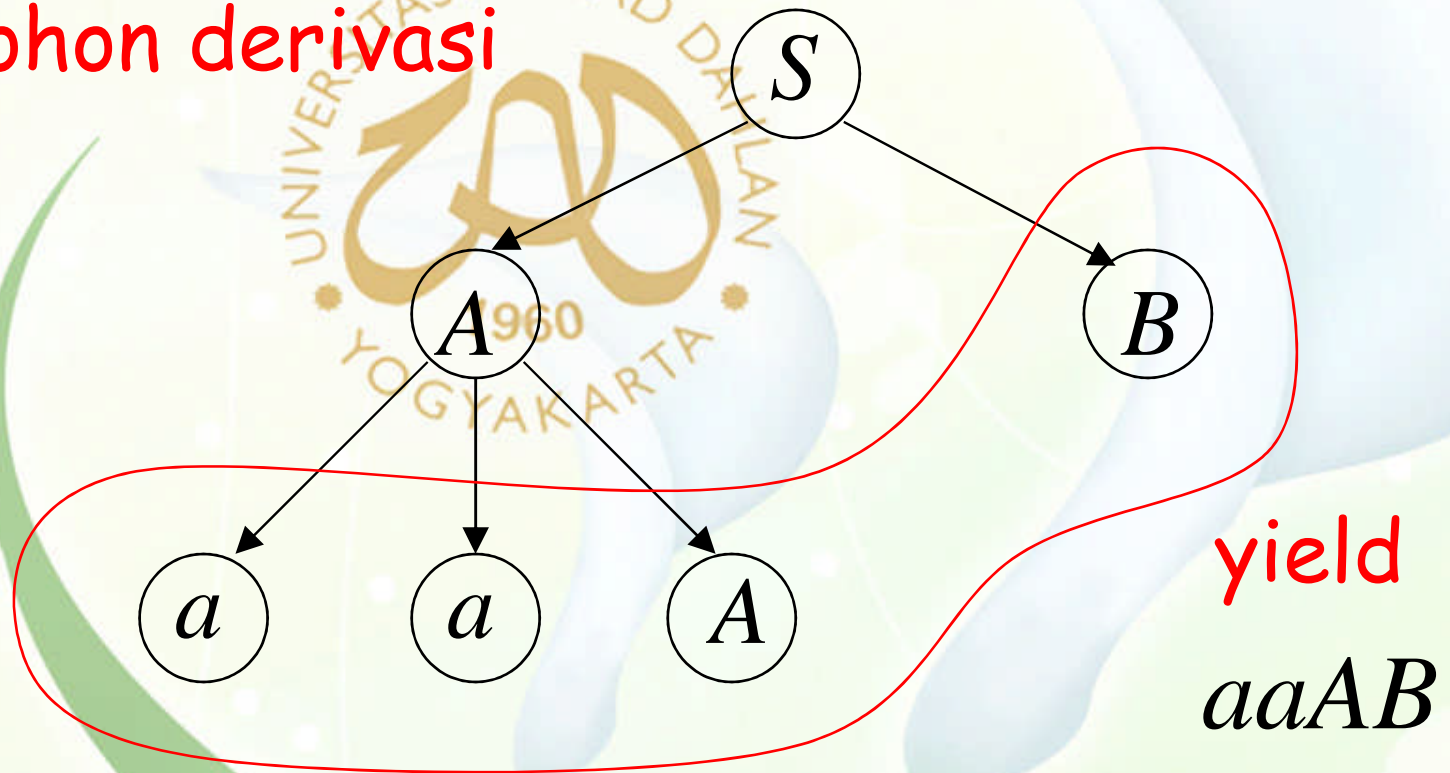
Parsial pohon derivasi



$$S \Rightarrow AB \Rightarrow aaAB$$

form
sentensial

Parsial pohon derivasi



Tidak masalah derivasi yang akan di pakai

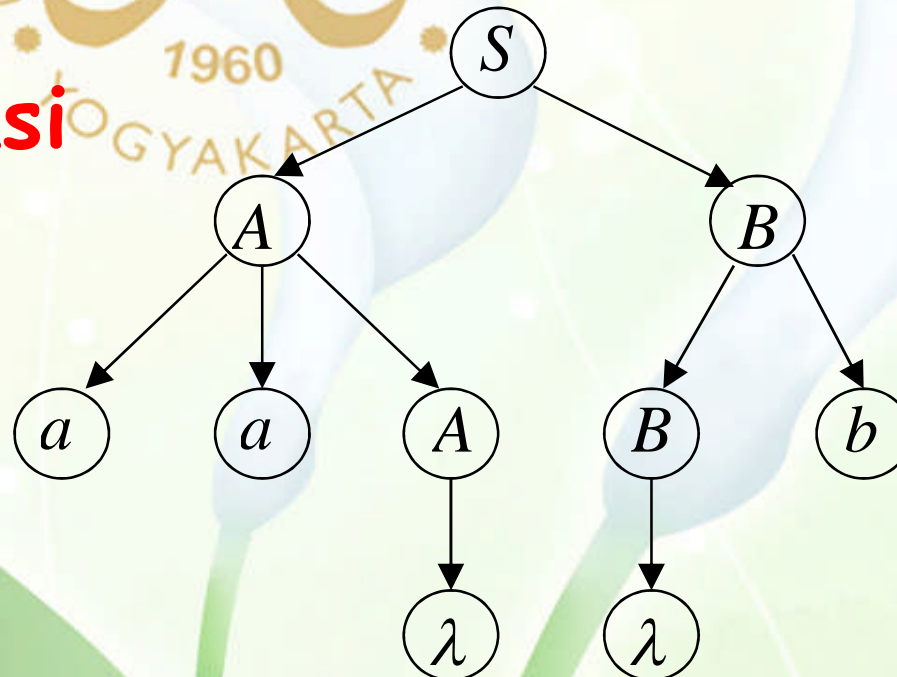
Kiri :

$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaB \Rightarrow aaBb \Rightarrow aab$

kanan:

$S \Rightarrow AB \Rightarrow ABb \Rightarrow Ab \Rightarrow aaAb \Rightarrow aab$

Pohon derivasi

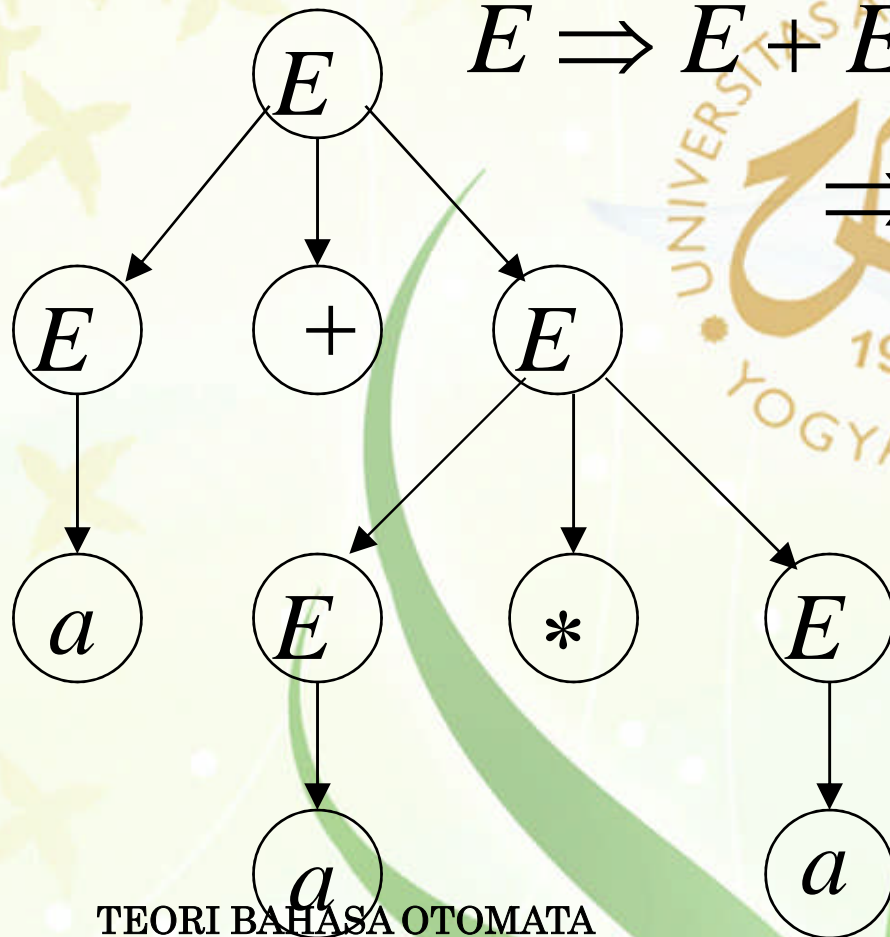




Ambiguiti

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$



$$\begin{aligned} E &\Rightarrow E + E \Rightarrow a + E \Rightarrow a + E * E \\ &\Rightarrow a + a * E \Rightarrow a + a * a \end{aligned}$$

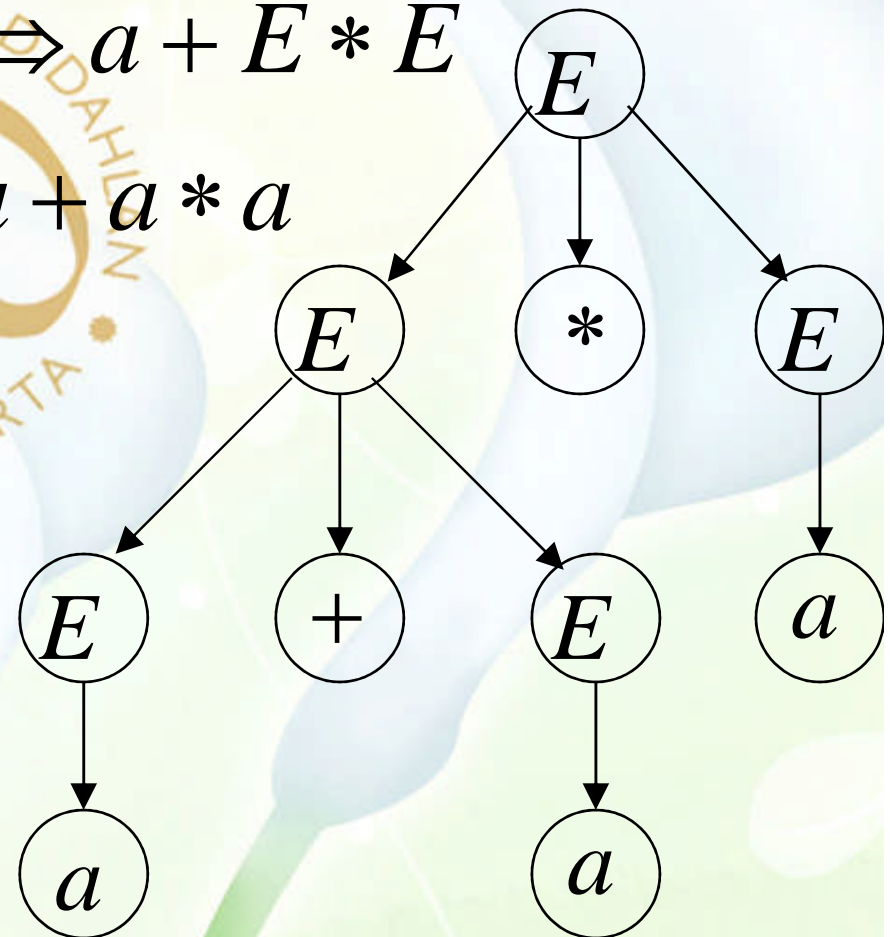
Derivasi kiri

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$

$$E \Rightarrow E * E \Rightarrow E + E * E \Rightarrow a + E * E \\ \Rightarrow a + a * E \Rightarrow a + a * a$$

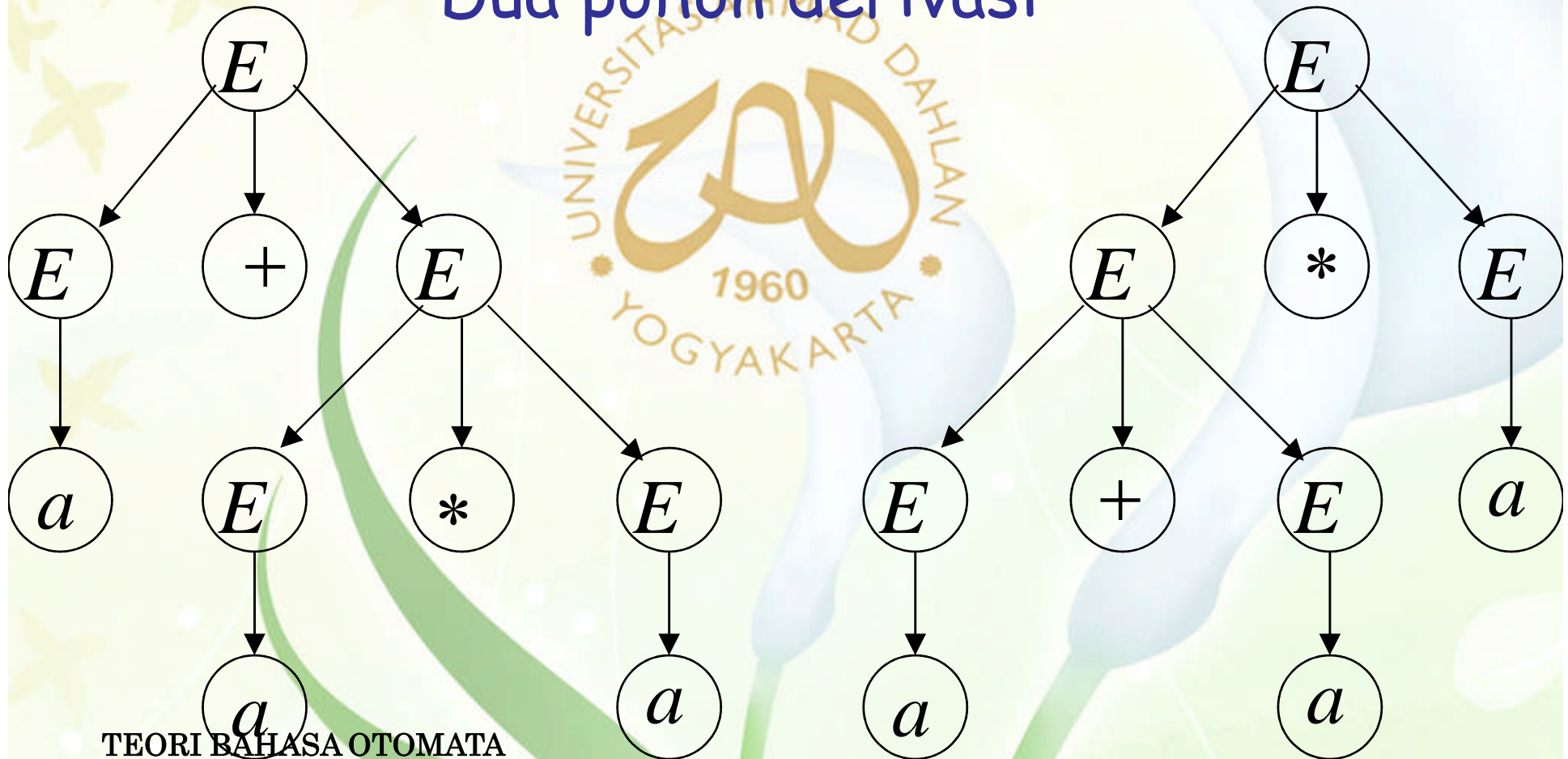
Derivasi kiri



$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$

Dua pohon derivasi

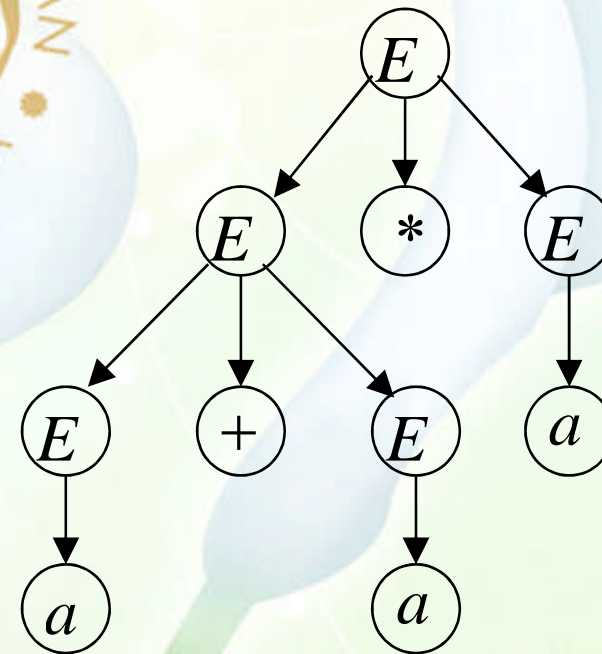
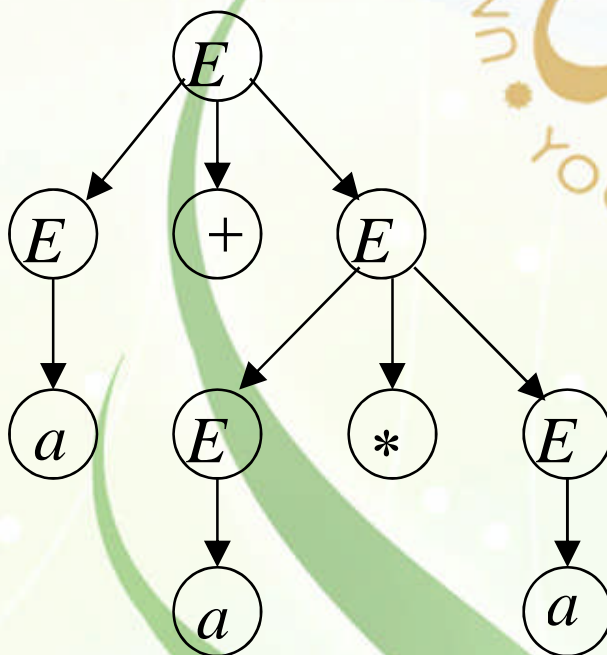


Grammar

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

Adalah **ambiguus**:

string $a + a * a$ Mempunyai dua pohon derivasi



Grammar $E \rightarrow E + E \mid E * E \mid (E) \mid a$

Adalah **ambiguus**:

string $a + a * a$ Mempunyai dua pohon derivasi

$$\begin{aligned} E &\Rightarrow E + E \Rightarrow a + E \Rightarrow a + E * E \\ &\Rightarrow a + a * E \Rightarrow a + a * a \end{aligned}$$

$$\begin{aligned} E &\Rightarrow E * E \Rightarrow E + E * E \Rightarrow a + E * E \\ &\Rightarrow a + a * E \Rightarrow a + a * a \end{aligned}$$

Definisi:

Grammer bebas konteks G adalah **ambigius**

Jika beberapa string $w \in L(G)$ mempunyai :
dua atau lebih pohon derivasi

Dengan kalimat yg lain:

Grammer bebas kontek G adalah **ambigius**

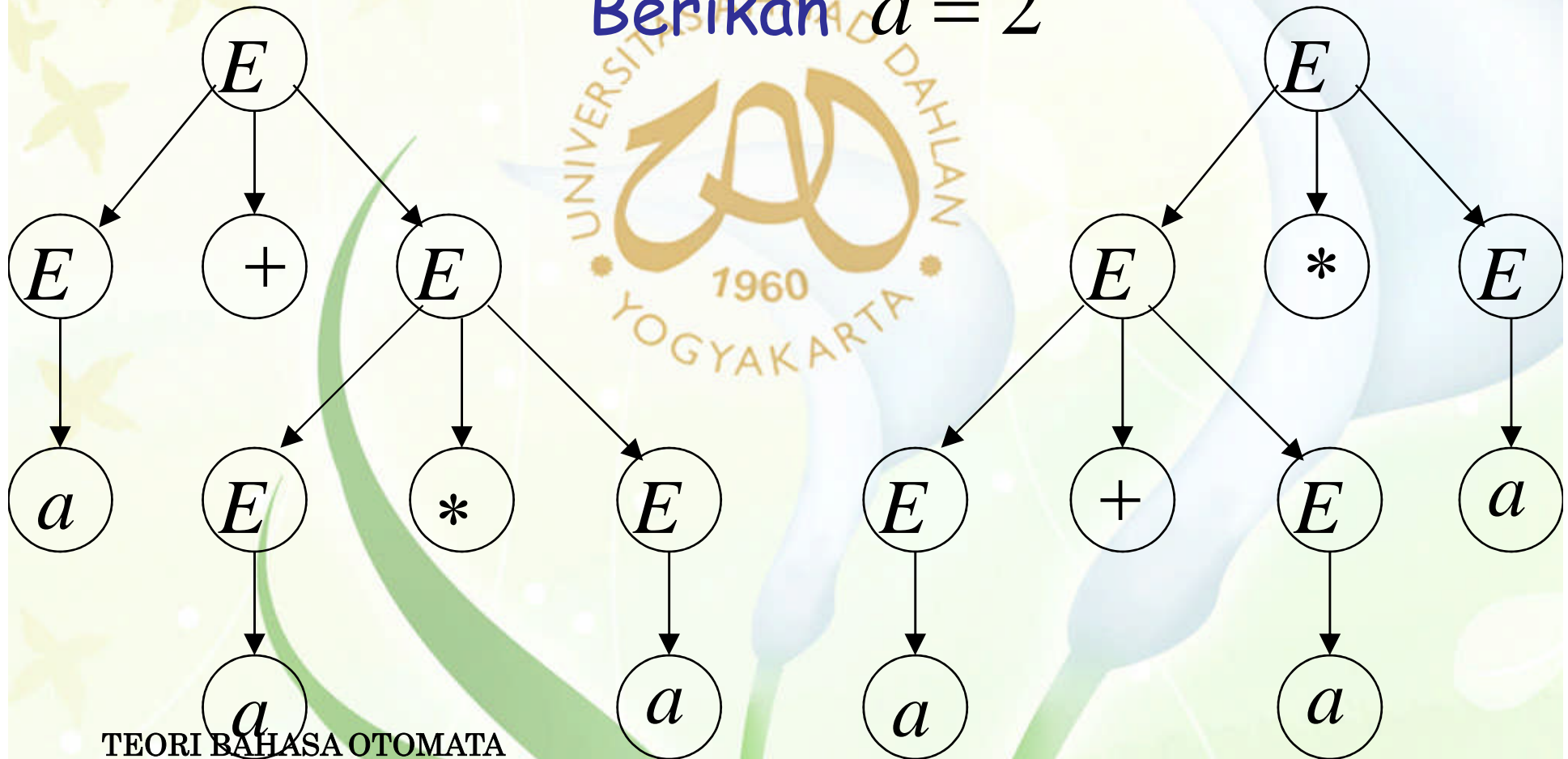
Jika beberapa string $w \in L(G)$ mempunyai:

dua atau lebih derivasi kiri
(atau kanan)

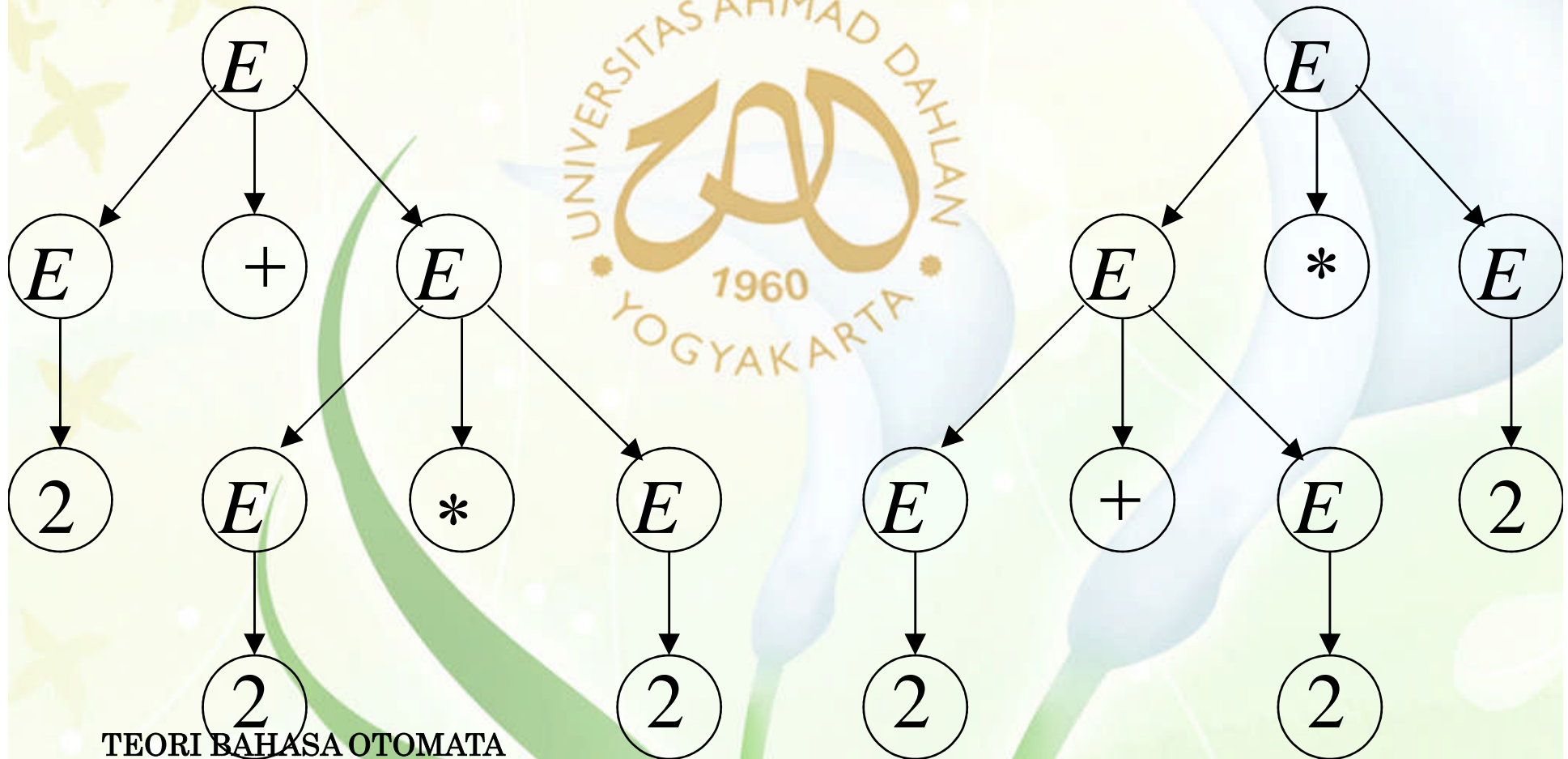
Bagaimana mengetahui ttg ambiguiti?

$$a + a * a$$

Berikan $a = 2$

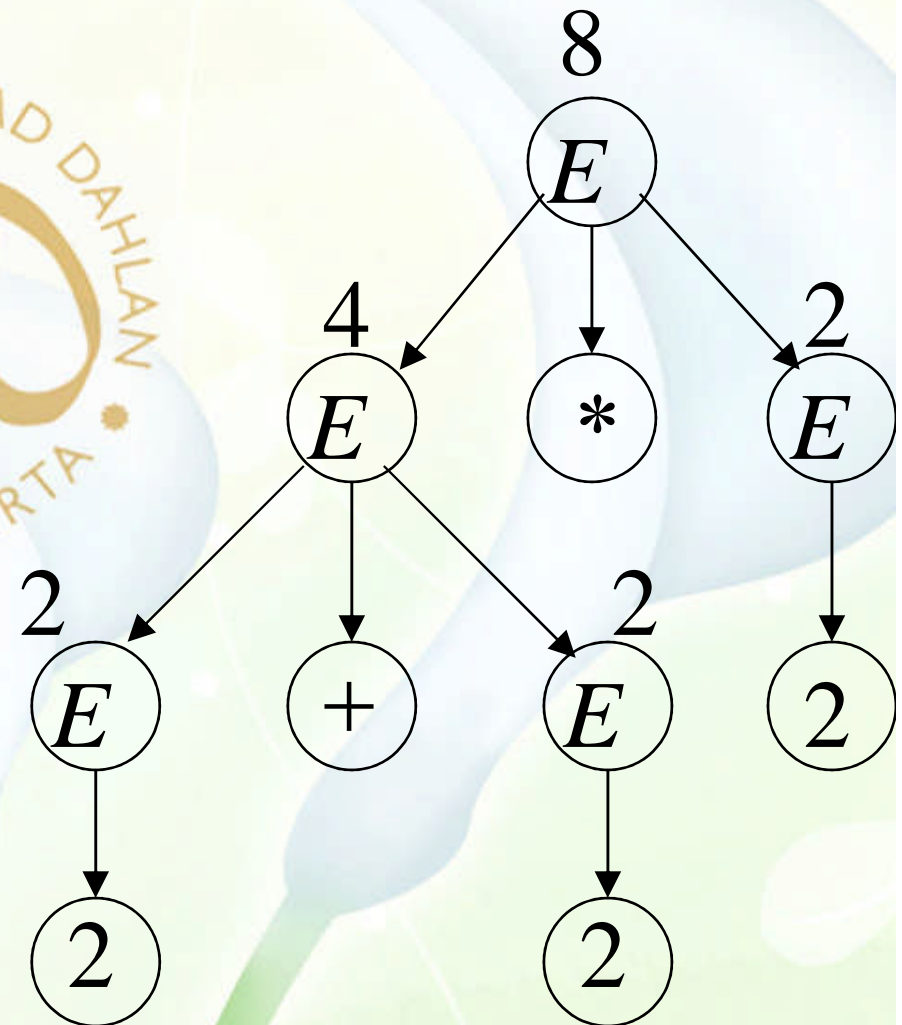
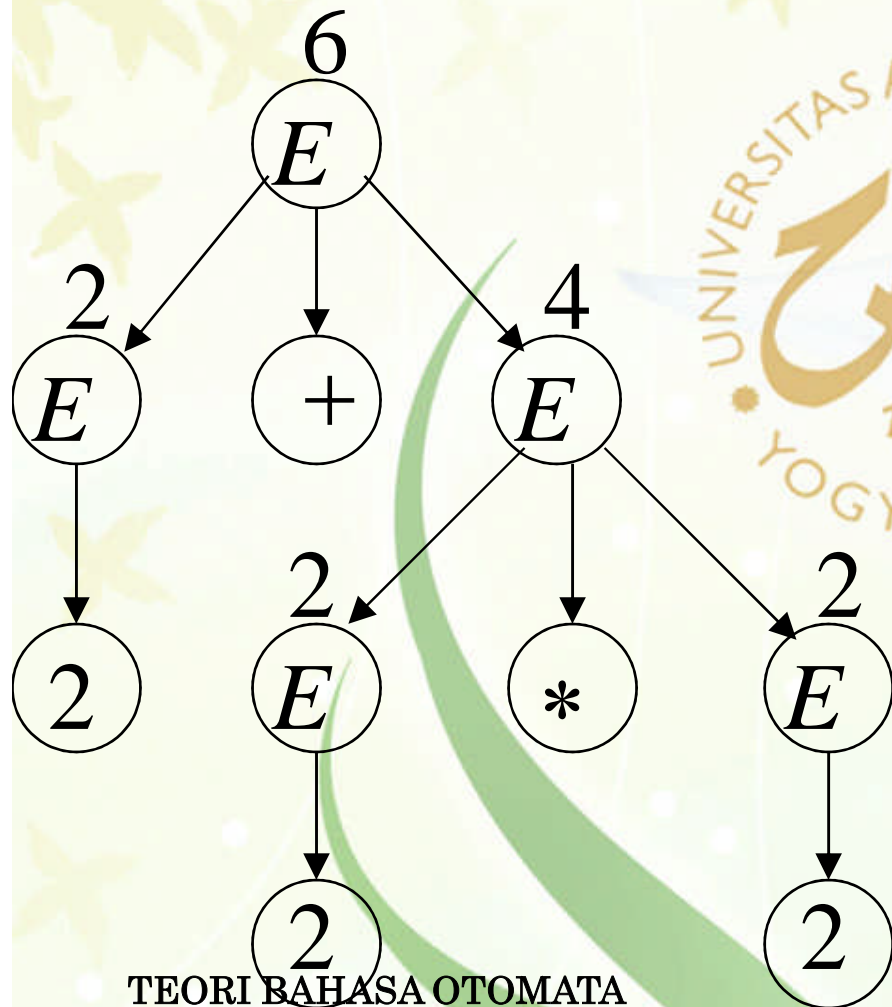


$$2 + 2 * 2$$

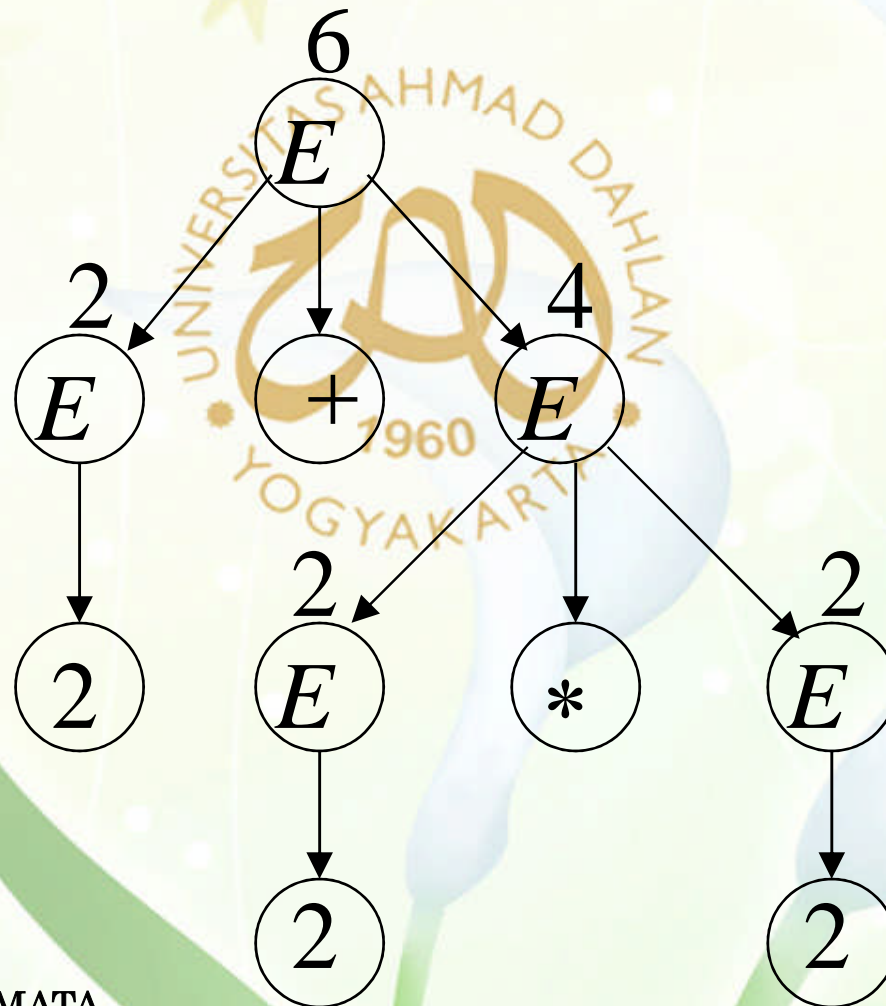


$$2 + 2 * 2 = 6$$

$$2 + 2 * 2 = 8$$



Hasi yg benar: $2 + 2 * 2 = 6$





Ambiguiti tidak baik untuk bahasa pemrograman

Membetukan **ambigius** gramer:

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

Gramer **non-ambiguous** baru :

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow a$$

$$E \Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + T * F$$

$$\Rightarrow a + F * F \Rightarrow a + a * F \Rightarrow a + a * a$$

$$E \rightarrow E + T$$

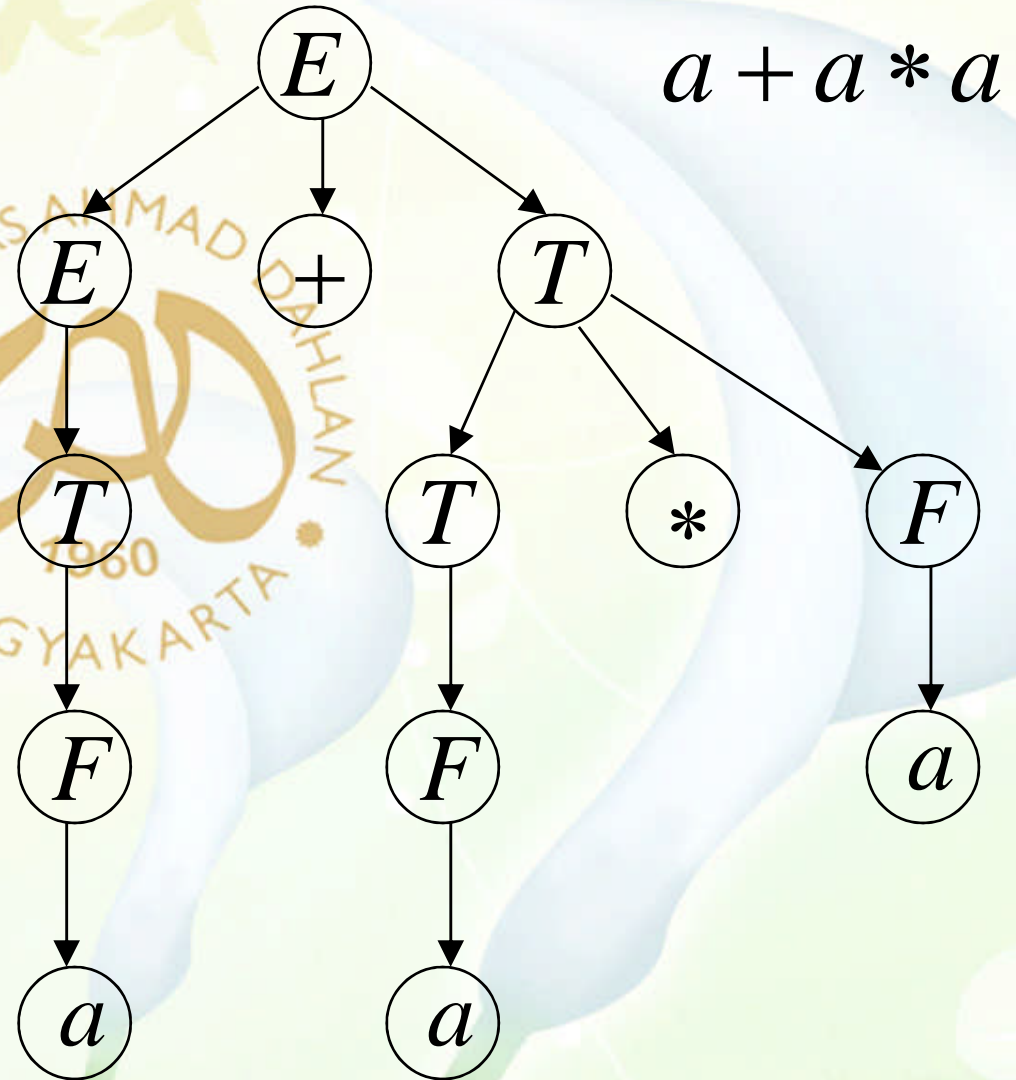
$$E \rightarrow T$$

$$T \rightarrow T * F$$

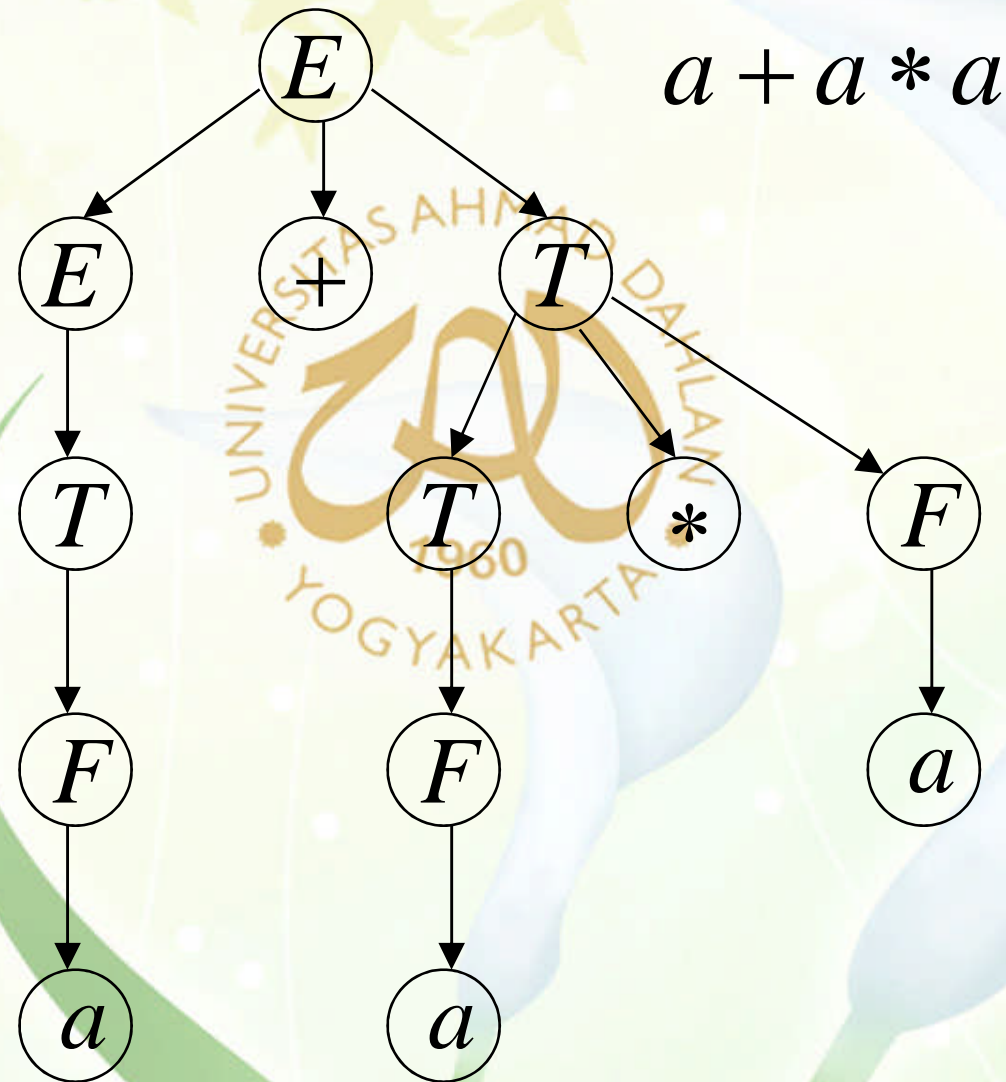
$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow a$$



Pohon derivasi yg unik



Grammer : $G \quad E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow a$

non-ambiguous :

Untuk setiap string $w \in L(G)$ mempunyai

Pohon derivasi yg unik

Pustaka

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